Geodynamics 5690/6690: Exercises 2 **Key**

Show all work; write clearly (Full sentences! Well-structured paragraphs!) in developing equations and discussing results.

(1) Which of the following are responsible for topographic elevation variations in the western United States?
(a) Temperature variations within the lithospheric thermal boundary layer
(b) Variations in thickness of a buoyant crust
(c) Variations in rock-types found within the crustal column
(d) Temperature variations within the convecting asthenosphere
(e) Surface processes (including erosion, faulting and volcanic construction) and their isostatic response

[5] All of these are highly likely to contribute!

(2) Early in the semester, we read a paper by Kellogg et al. (1998) postulating two-layer mantle convection in which the deeper layer was a thin, compositionally-distinct zone at the bottom of the mantle (in what are now called Large Low Shear Velocity Provinces, or LLSVPs).

(a) Describe at least three hypotheses for the origin of LLSVPs.

[5] More than three possibilities have been hypothesized: These include i. ancient oceanic crust that hitched a ride to the lower mantle with subducted slab; ii. cumulates that differentiated during formation of continental crust, eclogitized and sank to the base of the mantle; iii. mantle from a Mars-sized planet (Theia) that collided with early Earth and sank within; iv. clusters of small plumes that are smeared by imaging; and (recently), v. separation of Fe^{3+}-rich and Fe-poor bridgmanite phases that may have occurred in cooling of molten mantle at lower-mantle P-T conditions (Wang et al., Nature 2021).

(b) Kellogg and other papers we discussed in class listed more than five observations supporting incomplete mixing of the mantle. Describe at least three of these observations, and for each one, discuss how it is or is not consistent with each of the three hypotheses you listed in part a.

[5] Kellogg et al. (1998) discussed (1) differences in isotopic chemistry of ocean island basalts (OIB) and mid-ocean ridge basalts (MORB); (2) crustal Sr and Nd isotopic ratios coupled with atmospheric ^{40}Ar that suggest that much of the mantle has been outgassed and melt depleted, but (3) mantle Xe, He and Ar isotopes indicating that much of the mantle is still enriched in these primordial gases; and (4) surface heat flow arguments implying an undepleted deep reservoir for radioactive heat producing elements. Among the hypotheses: i. Oceanic crust would be slightly enriched in K, U and Th but mostly MORB chemistry and empty of primordial gases, so unlikely. ii. Continental-derived cumulates have an OIB-consistent chemistry but would be less enriched in radioactive elements than basalt, and empty of noble gases. iii. Theia mantle likely would be enriched in radioactive elements and gases (as it collided too early to off-gas or melt-deplete), but it is unclear whether it would have OIB-enriched chemistry. iv. There is no reason to

anticipate that plumelets would be substantially compositionally different than the rest of a mixed mantle. And, v. Bridgmanite phase separation would likely produce compositional differences, but Wang et al. (2021) did not address whether/why it would concentrate OIB isotopes, radioactive elements or noble gases in the iron-rich component.

(c) Given the observations, which hypothesis you find most plausible and why?

[5] In addition to the mantle mixing arguments, a successful hypothesis should match the slightly-low shear wave velocity, significantly higher density (and thus iron-rich, slightly silica-rich) properties measured via seismic observations of LLSVPs. It should also be otherwise consistent with Earth-history and flow dynamics (as discussed in Kellogg et al. [1999]). Oceanic crust, continental cumulates and plumelets are unlikely by themselves due to their chemistries, but either the Theia or the bridgmanite separation hypotheses are (arguably) consistent with the observations (though Theia mantle seems a stronger candidate sans evidence for radioactive elements and noble gases separating into an iron-rich bridgmanite melt, given the compositional differences—high fayalite concentration, high silica—expected for the mantle of a Mars size body). It's not unreasonable though to expect that LLSVPs are an admixture of two or more of these hypothetical source mechanisms.

(3) Early seismic studies of the Sierra Nevada mountain range in California found no evidence for an isostatic "crustal root" that researchers expected to find (and partially wrote their proposals based on finding). Assuming a topographic amplitude of 3000 m, a regional effective elastic thickness of 15 km, ρ_c = 2670 kg/m^3, and ρ_m = 3350 kg/m^3:
(a) Assume that the ~100 km width of the range represents half the wavelength of sinusoidal flexural loading and response. What would be the amplitude of Moho deflection expected if the range is a surface load? (Remember: The final surface topography is the surface load ***plus*** the flexure.) What should the amplitude of Moho deflection be if the topography is a flexural response to loading at the Moho?

[10] For sinusoidal surface loading of amplitude H_I, flexure takes the form:

$$H = \frac{\Delta\rho\xi}{\rho_c + \Delta\rho\xi} H_I = \frac{\rho_m - \rho_c + \frac{D}{g}k^4}{\rho_m + \frac{D}{g}k^4} H_I$$

The final topography would be $H = H_I + W$, so $W = H - H_I$. Rearranging and solving for W,

$$W = -\frac{\rho_c}{\rho_m - \rho_c + \frac{D}{g}k^4} H$$

Flexural rigidity $D = ET_e^3/(12[1-\nu^2])$. Assuming a typical Young's modulus $E = 10^{11}$ Pa and Poisson's ratio $\nu = 0.25$, $D = 3\text{x}10^{22}$, and $k = 2\pi/\lambda = 3.14\text{x}10^{-5}\ \text{m}^{-1}$, so $W = -2.19$ km would be the Moho deflection.

If the topography were a flexural response to a load deflection of the Moho,

$$W = H = -\frac{\Delta\rho}{\Delta\rho + \rho_c\phi} W_I = -\frac{\rho_m - \rho_c}{\rho_m + \frac{D}{g}k^4} W_I \quad \Rightarrow \quad W_I = -\frac{\rho_m + \frac{D}{g}k^4}{\rho_m - \rho_c} H$$

and in that case the Moho amplitude is $W_I - W = -24.9$ km.

(b) Now assume that the ~400 km length of the range is half the wavelength of loading and response. What would the Moho deflections be in that case if it is a surface load? What would it be if it is a subsurface load?

[5] Given $\lambda = 800$ km, $k = 7.9\text{x}10^{-6}$ m^{-1} so $W = -11.6$ km for Moho if the Sierra Nevada is a surface load, and $W_I - W = -11.8$ km if it is a response to subsurface loading.

(c) The Sierra Nevada is two-dimensional, and we have discussed in class how 2D wavenumber is represented as a 1D k. What is the correct wavelength in this case, and what are the Moho deflections for surface and subsurface loading?

[5] The two-dimensional wavenumber is given by $k = \sqrt{k_1^2 + k_2^2} = 3.24\text{x}10^{-5}$, equivalent to a 194 km wavelength. In this case, $W = -1.98$ km for Moho if the Sierra Nevada is a surface load, and $W_I - W = -26.65$ km if it is a response to subsurface loading.

(d) Now look at the Moho deflection under the Sierra Nevada measured in more recent analyses by Lowry & Pérez-Gussinyé (Nature 2011). How does this compare to your calculations? What other dynamics are going on here that we've discussed in class, and how might that affect your interpretation?

[10] Visual inspection of Figure 4b in Lowry & Pérez-Gussinyé suggests that crustal thickness varies from ~30-35 km just to the east and west of the Sierra Nevada range to ~40-45 km at the peak of crustal thickness, suggesting an amplitude (i.e., half the peak-to-peak variation) of about 5-10 km. Since the calculation in (c) is most relevant here, this is significantly more than would be expected if the Sierra Nevada were a pure surface load, but less than we would anticipate if it were a result of subsurface loading at the Moho. However, as we have discussed in class, the Sierra Nevada range is underlain by an ongoing drip or delamination event, in which dense mafic cumulate material that collected in the lower crust during Farallon subduction is now sinking into the mantle (e.g., Zandt et al., 2004). The crust likely has been thickened in response to the Rayleigh-Taylor instability associated with that cumulate body, and the total isostatic balance partially reflects the negative buoyancy associated with the drip.

(4) Most models of lake and postglacial rebound assume Maxwell (linear) viscoelasticity in a 1D Earth with just a few layers having uniform viscosity. For example, Karow & Hampel (Int J Earth Sci 2010) modeled effects of Bonneville rebound on Wasatch fault strain using an Earth model with lower-crustal viscosity (15-30 km depth) of 10^{22} Pa s, upper mantle viscosity (30-100 km depth) of 10^{18} Pa s, and asthenospheric viscosity (> 100 km) 10^{17} Pa s (based on results of earlier models that had parameterized layer thicknesses and viscosities by fitting the observed shoreline uplift).

(a) Karow & Hampel (2009) infer that their 10^{18} Pa s upper mantle layer corresponds to the mantle lithosphere, because Zandt et al. (1995) teleseismic data suggest a base of lithosphere at ~60-70 km (similar to Levander & Miller, 2012) and basaltic melts originate at similar depths. How do each of these observations relate to the various definitions of "lithosphere" that we have discussed in class (hint: go back to the beginning!), and more specifically how do these other definitions relate to the rheological definition of lithosphere?

[5] Like K&H, earlier papers modeling Bonneville & Lahontan rebound (e.g., Bills et al., 1994; 2007) were simple elastic layer over viscoelastic half-space representations of rebound (but with a few extra layers of varying viscosity that were poorly constrained by the available data). If we recall that for a half-space $w = w_0 \exp(-t/\tau)$ and viscosity η relates to decay timescale τ as:

$$\tau = \frac{4\pi\eta}{\rho g \lambda},$$

then on the time-/space-scales represented by the data (<300 km wavelengths and $\leq$17.5 kyr), a 10^{22} Pa s viscosity would have $\tau \geq 404$ kyr while a 10^{18} Pa s viscosity would have timescales $\tau \geq 40$ years. In the former case the modern-day response to Bonneville unloading would be < 4% complete, while in the latter case the response would have finished a long, long time ago. The layering complicates the picture slightly but for all intents and purposes, from an isostatic response perspective the lower-crustal viscosity would be considered part of the "lithosphere" while the upper mantle can be considered "asthenosphere" (and a very runny asthenosphere at that).

The "seismic LAB" impedance contrast imaged by Zandt et al. (1994) fits the "seismic lid" definition of lithosphere and probably relates to partial melt or water in the mantle, as discussed in class. The correlation of Basin & Range seismic LAB with the source depth of partial melts may be an indication of the nature of the boundary, but the correspondence of the seismic LAB with the base of Bills et al.'s 10^{18} Pa s viscosity upper mantle is just coincidence: There really isn't enough information content in Bonneville shorelines to constrain a layer boundary there.

(b) The Moho temperature under Bonneville (Schutt et al., 2018) averages about 700°C at ~31 km depth. Using the dislocation creep power law parameters given below (from Bürgmann & Dresen, 2008), and assuming a water fugacity of 5800 Pa (the approximate saturation fugacity for olivine at that depth), first calculate the flow yield strength that crustal and mantle rocks should have given an assumed strain rate of 10^{-15} s^{-1}; then calculate the strain rates that would be required (for both wet and dry rocks) to get an effective viscosity ($\eta_{eff} = \Delta\sigma / 2\dot{\varepsilon}$) of 10^{22} in the lower crust and 10^{18} in the mantle:

	Pre-exponential coefficient A	Water fugacity exponent r	Exponent n	Activation energy Q	Activation volume V
wet anorthite	1.58	1	3	3.45×10^5	3.8×10^{-5}
dry anorthite	5.01×10^{12}	0	3	6.41×10^5	2.4×10^{-5}
wet olivine	1.58×10^3	1.2	3.5	5.2×10^5	2.2×10^{-5}
dry olivine	10^5	0	3.5	5.3×10^5	1.8×10^{-5}

(Hint: If you aren't sure of your calculations, you'll be able to check them by comparing to yield strengths that you can calculate using computer codes you'll use to answer question 5). Assume a mean crustal density of 2800 kg/m^3; the grain-size exponent m for dislocation creep is zero; and the units of A will yield $\Delta\sigma$ in MPa (so multiply by 10^6 to get mks units of Pa). Typical surface strain rates are in the range 10^{-14} to 10^{-16} and strain rates at the Moho may be several orders of magnitude higher. What does this imply about the viscosities assumed in Karow & Hampel?

[10] The dislocation creep law can be expressed as $\Delta\sigma = \left(\frac{\dot{\varepsilon}}{A} f_{H_2O}^{-r}\right)^{1/n} \exp\left(\frac{Q+PV}{nRT}\right)$. Plugging in the numbers above, assuming $P = \rho g h = 851$ MPa for crustal density $\rho = 2800$ kg m^{-3}, and noting that $T = 700$°C will be 973°K, at 10^{-15} s^{-1} strain rate the strengths are 2.7 MPa for wet anorthite, 400.3 MPa for dry anorthite, 59.3 MPa for wet olivine, and 446.5 MPa for dry olivine. The corresponding effective viscosities are $1.4\text{x}10^{21}$ Pa s for wet anorthite, $2.0\text{x}10^{23}$ Pa s for dry anorthite, $3.0\text{x}10^{22}$ Pa s for wet olivine, and $2.2\text{x}10^{23}$ Pa s for dry olivine.

To solve for the strain rate that matches a given effective viscosity, we plug the constitutive law into the effective viscosity relation. This yields:

$$\dot{\varepsilon} = \left(\frac{10^6}{2\eta_{eff}}\right)^{n/n-1} \left(\frac{f_{H_2O}^{-r}}{A}\right)^{1/n-1} \exp\left(\frac{Q+PV}{(n-1)RT}\right).$$

It's important in this context to remember that A is given for stress in MPa, so to get correct units we must divide η_{eff} by 10^6 to get it in units of MPa s (thus the 10^6 factor in the numerator of the first term). So to get a 10^{22} Pa s viscosity in the crust one would need a strain rate of $5.0\text{x}10^{-17}$ s^{-1} in wet anorthite or $9.0\text{x}10^{-14}$ s^{-1} in dry anorthite, and to get 10^{18} Pa s viscosity in the uppermost mantle one would need strain rates of $1.8\text{x}10^{-9}$ s^{-1} in wet olivine or $3.1\text{x}10^{-8}$ s^{-1} in dry olivine.

Thus, one could reasonably get Karow & Hampel's lower crustal viscosity if flow is dominated by ~dry feldspar at reasonable Moho strain rates, but one would never expect olivine to have their upper mantle viscosity at the observed Moho temperature (the range of strain rates required would never occur in the lithosphere, and if anything the upper mantle strain rate should be less than that of the lower crust). The point of course is that a feldspar lithology is weaker than an olivine lithology, so any realistic layering of viscosity should take that into account and assign a higher viscosity to the uppermost mantle than that of the lowermost crust.

(c) The viscosity structure in Karow & Hampel was chosen based on earlier viscoelastic modeling of uplifted Bonneville shorelines. Think carefully about the modeling results in Willett et al. (1985) and the materials we discussed in class on the relationship of T_e to rheology. What limitations might one expect in estimating viscosities of a four-layer model from the forward modeling of uplifted shorelines as the response of a four-layer Earth with uniform viscosity in each layer? Hint: Nakiboglu & Lambeck (JGR 1983) discuss this also in an early paper modeling Bonneville rebound.

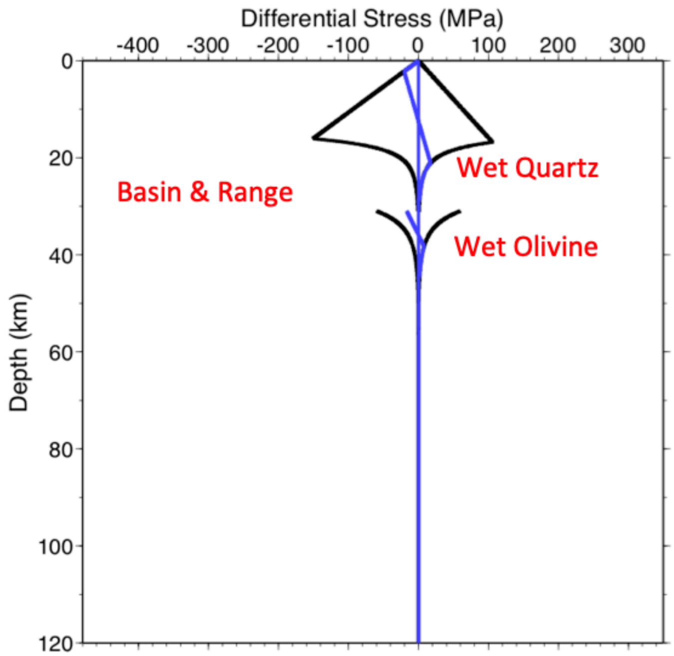
[5] There are several issues that one might consider, but one of the most important is that to assign thicknesses and viscosities in a four-layer Earth one would need enough information content to constrain at least three different viscosities. The spatial expression of vertical isostatic response in a layered medium for a snapshot in time will look almost exactly like that of a thin plate over an inviscid fluid (e.g., the Willett et al. papers and also Zhong JGR 1996), so all of the information about viscosity of the three layers must come

from sampling at independent times that differ by of order the relaxation time τ. We have deformation of three shorelines (Bonneville, Provo and Gilbert) that are sampled at much longer intervals than the relaxation times of the deepest two layers inferred by Bills et al. (with 10^{18} and 10^{17} Pa s viscosities). Nakiboglu & Lambeck noted that layered models are very insensitive to the presence of a thin weak layer that is sandwiched between two stronger layers (e.g., the lower crust). This is related to the similarity of uplift response of two decoupled layers to that of a single, stronger layer as described in Willett et al. Most importantly, the parameter space could be greatly reduced (from three unknown thicknesses and three unknown viscosities) by modeling two layers (crust and mantle) with known thickness, known temperature and flow laws that could then be parameterized in terms of one variable (e.g., water fugacity!)

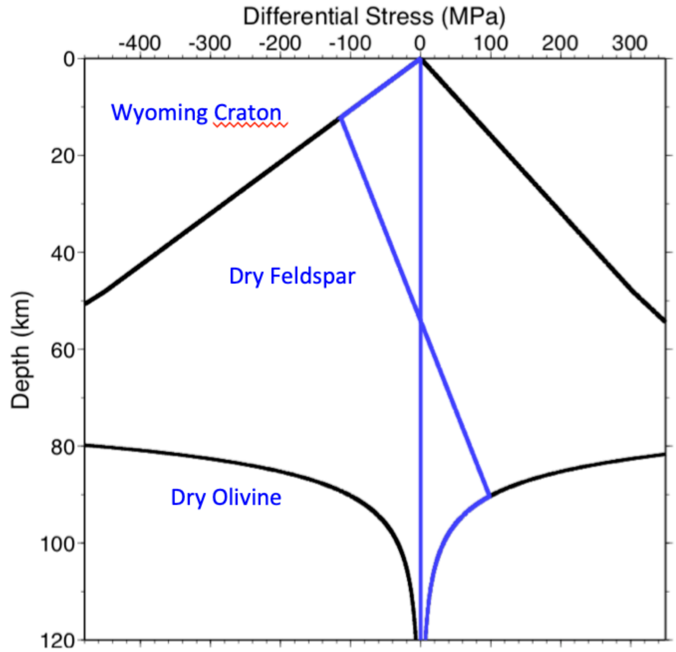
For exercise (5), you will need to run a fortran code that I have created for you and placed in the course Canvas website (under the "Files" heading). This exercise will be easiest to do if you use a Mac computer with Matlab installed (it should be possible to compile and run the code on other operating systems also, but I will only give instructions for Mac unix here). You'll need to open a terminal in order to run and/or compile the codes. If the zip file linked to these exercises did not automatically unpack you can go to the directory where you put it (e.g.,
cd Desktop/
and unzip by command:
unzip G5690.zip
cd G5690
If your Mac OS is the same as mine you may be able to run the executables that are already in that directory & I would try that first:
./YSE_Plot
If the executable is not compatible with your OS, then
cd Source/
sh fcomp.sh
cd ../
and if you have gfortran on your machine as your fortran compiler, you should be ready to go. (If not, you may need to edit fcomp.sh and replace "gfortran" with "gcc" or whatever your fortran compiler is). Output files from these codes can be plotted in Matlab using
YSE_Plot
with output figures printed to a file in png format. (GMT shellscripts to do plotting are also included, but they are written in GMT v4.5, which is not commonly used anymore).

(5) Use the code YSE_Plot to model geotherms, yield strength envelopes, effective viscosity and T_e.
(a) First, use the geotherm parameters derived for Exercise 1 question 5d to specify a geotherm. (You can use the parameters I specified in the Key posted on the website to match observations of the Moho temperature and surface heat flow.) Effective elastic thickness in the Bonneville region is about 10 km, while in the Wyoming craton it's nearer 90 km. Which (if any) rheological layering combinations most closely match these two cases if you use the geotherms for these regions that mathch the observations? Plot the corresponding yield strength envelopes for each location using YSE_Plot in Matlab. Do these match what you might expect to see for this region?

[10] Using the geotherm parameters for the Basin & Range with a wet quartz over wet olivine profile and a strain rate of 10^{-15} s^{-1} gives a T_e of 11.9 km (and using 10^{-16} s^{-1} strain rate knocks it down further to 10.7 km). The next-weakest lithology, wet feldspar over wet olivine, gives a $T_e = 15.7$ km even with very low strain rate.

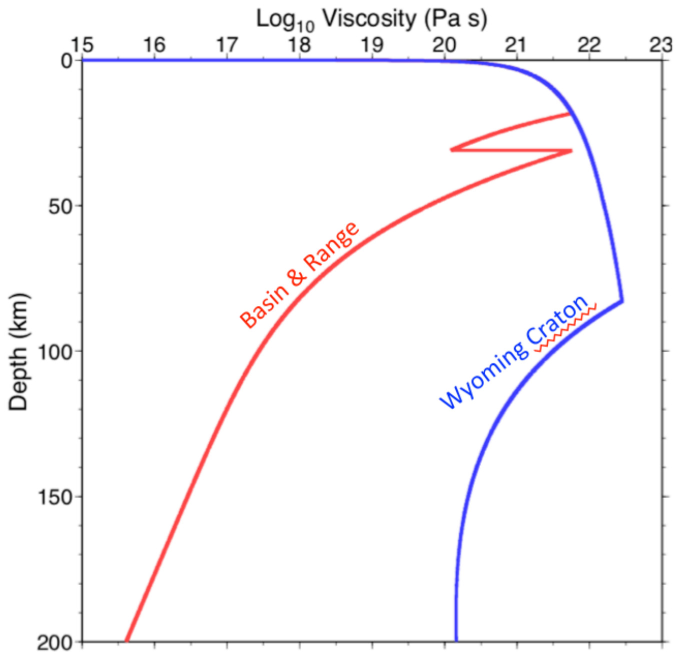


For the Wyoming craton, either dry feldspar over dry olivine or dry pyroxene over dry olivine give the same answer of a strongest possible end-member. For a 10^{-15} s^{-1} strain rate they predict an 85.9 km isotropic T_e (with max $T_e = 93.2$ km in the absence of membrane stress). Such a high (10^{-15} s^{-1}) strain rate is less likely for a strong lithosphere, and the model T_e would be lower for lower strain rates (e.g., 80.5 km isotropic and 86.8 km max for 10^{-16} s^{-1}). However, both crustal lithologies are in the brittle-field at the Moho, yielding a single welded plate, and there is no stronger lithology likely for the mantle than dry olivine.



(b) YSE_Plot also creates a file called EffVisc.zh containing the effective viscosity for a given yield strength envelope. Run the code once using your preferred Basin & Range parameters and rename that file to call it EffVisc_BR.zh; then run it again with Colorado Plateau parameters and rename the output EffVisc_CP.zh. Then you can use Visco in Matlab to plot viscosity with depth for both locations. Does the Bonneville region viscosity match that modeled by Karow & Hampel based on earlier rebound studies if you use the suggested strain rate? Note the strength and viscosity calculations used here depend on the assumed strain rate. Is a constant strain rate a reasonable approximation to make? How does it change if you change the strain rate? What does that imply about the rebound process?

[10] Using the suggested 10^{-14} s^{-1} strain rate to calculate viscosities produces the profiles plotted below. The upper crust was elastic in Karow & Hampel's model, whereas ours is $< 10^{22}$ Pa s because brittle-field rheological strength was used to define viscosity there. (The Yield Strength Envelope actually assumes an elastic-plastic rheology in the brittle-field though so "viscosity" is a misnomer, but elasticity is also not correct). Viscosity of the lower crust is much lower than their 10^{22} Pa s, ranging from 1×10^{20} to 4×10^{21} Pa s. The upper mantle ranges from 10^{15} to 6×10^{21} Pa s with "average" $\sim 10^{18}$, so roughly similar to their 10^{18} Pa s parameterization. This is roughly consistent with the modeling of Bills et al. (JGR 1994) that was the source for the Karow & Hampel model parameters, because the latter modeling has poor depth resolution and likely will be most sensitive to the weakest rheology.



Based on our calculations for question 4, one can readily show that strength increases proportional to $\dot{\varepsilon}^{1/n}$ but viscosity decreases proportional to $\dot{\varepsilon}^{1-n/n}$. A constant strain rate versus depth is not a realistic assumption; in fact strain rate will depend on deviatoric stress (higher stress = higher strain rate) and viscosity. The highest strain rates would be expected where strength is lowest (i.e., in the lower crust and deeper in the mantle) and where deviatoric stresses are highest (e.g., at the Moho where crustal thickness is changing near the Wasatch front). Strain rates are also perturbed by the lake unloading and those rates would have been highest (and correspondingly, viscosities would be lowest) at times immediately following large, rapid changes in the load (e.g., following the Bonneville flood and drop to Provo level).

(6) Based on the results of these calculations, what can you infer about controls on deformation processes in the western United States?

[5] The data certainly suggest that temperature variations are significant in determining rheology (and thus strength and deformation) of the Cordillera, but as discussed in class, lithology variations and especially water have very important roles to play. Water seems to be moved around in the mantle by subduction-related processes primarily, and melt flux no doubt has an important role in transferring water up the lithospheric column. Partial melt itself has a rheological expression, reducing the viscosity by several orders of magnitude, and likely plays an important role as well.