Geodynamics 5690/6690: Exercises I **Due 3 March**
Show all work; your writeup should include answers to all questions and any relevant figures you create.

(1) Roy et al. (2009)'s "preferred" geotherm model assumed steady-state initial conditions (i.e., for $t \leq 0$) including a surface temperature of 0°C and a basal temperature (at 200 km depth) of T_a = 400°C. They don't mention thermal conductivity, but their thermal diffusivity κ = 10^{-6} implies a constant thermal conductivity ~ 3 W/m°K.
a. Given these parameters, what would be the surface heat flow at time $t \leq 0$?

[5] Heat flow q is given by $q = -k\frac{\mathrm{d}T}{\mathrm{d}z}$. For constant k and no heat production, q will be constant through the lithosphere (including at the surface), and thus we require:
$q_s = -k(\Delta T)/(\Delta z)$ = –(3 W/m°K) X (–400 °K) / (200,000 m) = 6 mW/m^2.

b. Roy et al. also assumes heat production of 3x10^{-6} W/m^3 in a layer from 0 to 15 km depth. What would the heating contribute to surface heat flow? What temperature change would be expected across this layer just due to radioactive heating?

[5] Heat flow changes via heat production as $\frac{dq}{dz} = A$, so the total change in heat flow (and hence the contribution to surface heat flow by the radioactive layer) is
$\Delta q = A\Delta z$ = (3x10^{-6} W/m^3) X (15000 m) = 45 mW/m^2.
The geothermal gradient will be slightly higher in the heat-producing layer than it would be without heat production because of the incremental addition of heat, and the temperature change ΔT due to radiogenic heat production alone, over the thickness of the heat producing layer Δz, will be
$\int dT = -\frac{1}{k}\int q(z)dz$ = – (15,000 m) X (0.045 W/m^2) / (2 • 3 W/m°K) = –112.5 °K

c. The total geotherm is the sum of the effects from conduction plus those from radioactive heating in the 15-km thick layer. With the radioactive heating present, the conduction part occurs across a 200-km thick layer with temperature change of 400°C minus the temperature change calculated in part b. What is the total surface heat flow for this geotherm?

[5] With the heat-producing layer, the temperature change between 0 and 200 km depth from linear conduction of the basal heat flow q_0 is –(400 – 112.5) = –287.5 °K, and
$q_0 = -k\Delta T/\Delta z$ = –(3 W/m°K) X (–287.5 °K) / (200000 m) = 4.3 mW/m^2.
The surface heat flow is the sum of the basal heat flow plus radiogenic heat flow, or 49.3 mW/m^2.

(2) At time t = 0, the base of the tectosphere (200 km deep under the Colorado Plateau; 100 km deep elsewhere) is assumed to be replaced by asthenospheric mantle with a temperature of T_b = 1300°C. Using their coefficient of thermal expansion (α = 2.5x10^{-5})

and a reference mantle density of ρ_0 = 3350 kg/m^3 for T_0 = 0°C, what instantaneous elevation change would be expected off the Plateau? (Assume Airy isostasy: i.e., uniform stress in a fluid asthenosphere and no flexural rigidity, so $\rho_0 g \Delta h = \int_{200}^{0} \Delta\rho g \, \mathrm{d}z$, where Δh is change in elevation and $\Delta\rho$ is change in density. Assume also that the geotherm calculated in question 1c is the same both on and off the Plateau!)

[5] Off the plateau, temperatures between 100 and 200 km instantaneously jump to T_b. Hence the new topography must balance a mass-per-unit-area change of $\int_{200}^{100} \rho_0 \alpha \Delta T \mathrm{d}z$.

Temperature at t < 0 is 400°C at 200 km and
$\mathrm{d}T/\mathrm{d}z = -q_0/k$ = –(4.3 mW/m^2) / (3 W/m°K) = –1.4x10^{-3} °C/m, so
T = 112.5 + 1.4x10^{-3}z °C between 100 and 200 km, and ΔT = 1187.5 – 1.4x10^{-3}z °C. Hence the mass-per-unit-area change is $\Delta\sigma = \rho_0\alpha(1187.5z - 0.0014z^2)|_{200}^{100}$ = (3350 kg/m^3) X (2.5x10^{-5} °C^{-1}) X (7.675x10^7 °C m) = 6.4x10^6 kg/m^2. In Airy isostasy (i.e., no horizontal stress), this must be balanced by an equal mass of topography $\rho_0\Delta h$, so
Δh = (6.4x10^6 kg/m^2) / (3350 kg/m^3) = 1.91 km (!)

(3) Assume the Laramide slab is 60 km thick and its temperature varies linearly from 400°C at the top to 1300°C at the bottom. If the slab is isostatically coupled to the base of the lithosphere for t < 0 and completely decoupled after, what would be the instantaneous elevation change everywhere at t = 0 due to delamination of the Laramide flat slab?

[5] If we arbitrarily set the top of slab as z = 0, ΔT = 900 – 0.015z °C from 0 to 60 km. Integrating, $\Delta\sigma = \rho_0\alpha(900z - 0.0075z^2)|_0^{60}$ = (3350 kg/m^3) X (2.5x10^{-5} °C^{-1}) X (2.7x10^7 °C m) = 2.26x10^6 kg/m^2. Hence Δh = (2.26x10^6 kg/m^2) / (3350 kg/m^3) = 675 m (and note this would be in addition to the amount calculated in question 2).

(4) Do these calculations suggest testable predictions for the hypothesis of Roy et al.? If so, what are the predictions, and what observations might you compare them with?

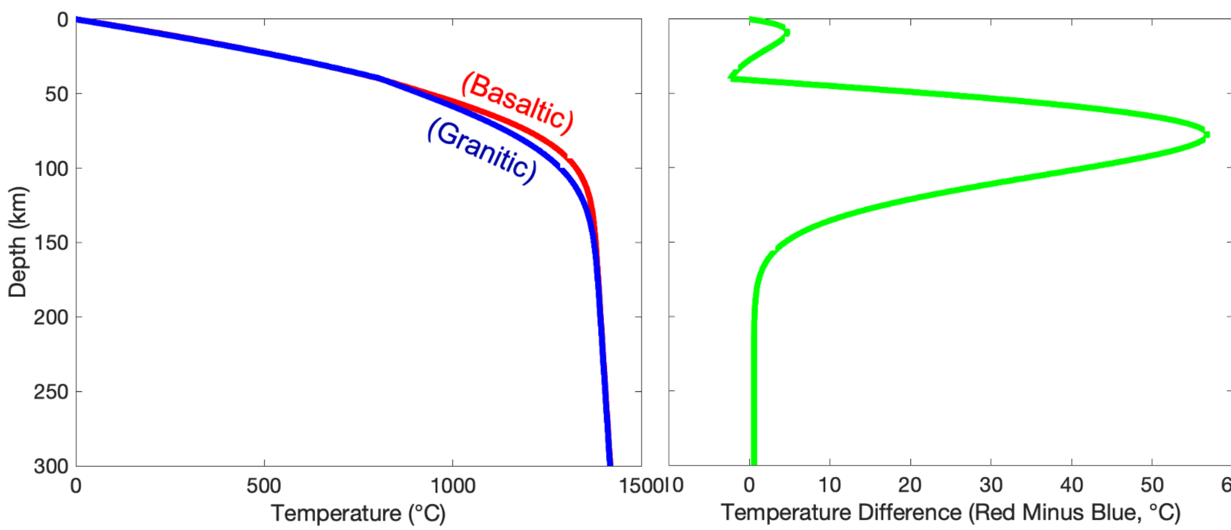
[5] Roy et al. (2009) neglected the elevation change that would be expected associated with the change from their $t < 0$ to $t = 0$ initial conditions. While one might reasonably argue that flexural strength of the lithosphere and flow associated with $t < 0$ flat slab subduction would both act to reduce slightly the height change relative to the 675 m (under the Colorado Plateau) and 1.9 to 2.6 km (outside) calculated by the Airy approximation, it is safe to assume that the sudden thermal uplift from sea level ~40 Ma would be large enough to be evident in the elevation record (and subsequent uplift would then have to be much smaller than calculated in the paper in order to finish with the modern elevation). Hence, their model oversimplifies (or over-complicates?) the actual history of thermal uplift. This may simply mean that the same model could be used with a higher temperature at 200 km and $t < 0$ to achieve the modern elevation, and indeed dynamical modeling suggests the base of the lithosphere would not cool below 800°C (and the lithosphere would cool only slightly in response to that during the ~20 Myr period of flat-

slab subduction). But there are other problems with this model (notably, the fact that modern heat flow and observed volcanism requires a large advective contribution that is not reflected in the Roy et al. conductive model, and melting at the temperatures assumed in their modeling requires hydration). Roy addressed dynamics of advection and flow more fully in subsequent papers that follow Roy et al. (2009)!

(5) There are several parameters in a conductive geotherm model that can affect both heat flow and temperature. You can explore the effects of these using the Matlab script Geotherm.m in Matlab, which is included as a download as part of this assignment. USU has a site-license for Matlab, so you should be able to either find it on Oldham-room computers or download it from the campus software site. You will need to place the Matlab script, Geotherm.m, in a directory where you want to work and then change your location to that directory using the file box within the software or from the Matlab command line using, e.g.,
>> cd /Users/~myusername/Desktop/
if that is where you put the script. Then you can run the code from the Matlab command line by typing:
>> Geotherm

Using the outputs from running the Geotherm script, discuss the effects on surface heat flow and Moho temperature for the following cases (with the remaining parameters held fixed to "Suggested values" [given in brackets]). As you discuss your results, be sure to specify what numbers you used!

(a) What are the surface heat flow and Moho temperature using thermal conductivity parameters expected for a mafic rock versus those for a quartz-rich rock?



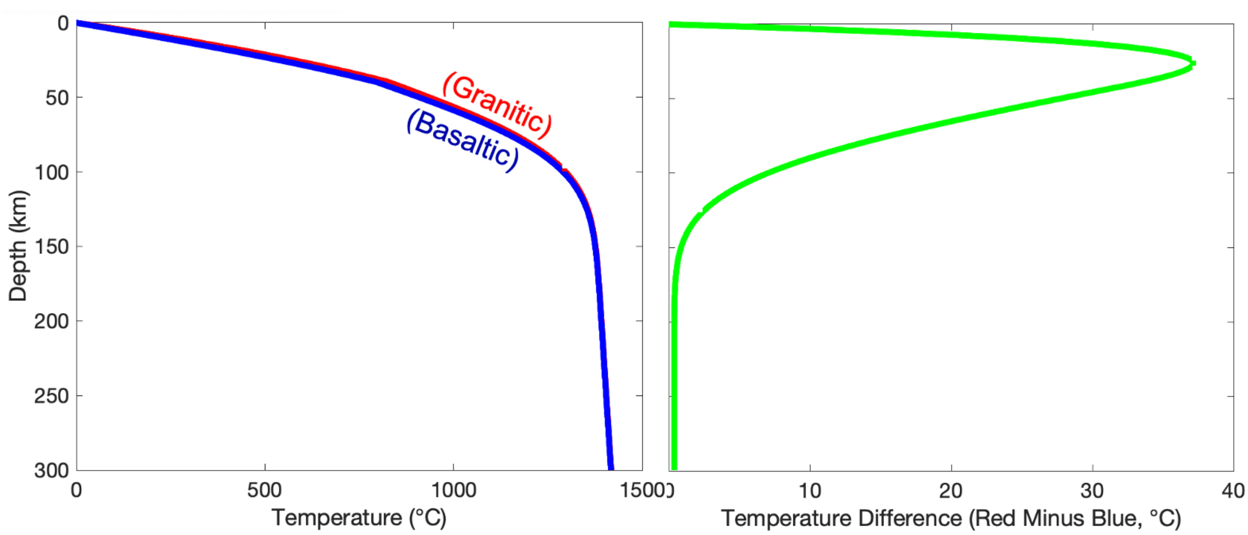
[5] A geotherm using the "suggested values" but with thermal conductivity parameters consistent with basalt ($A = 0.36$; $B = 1.4\text{x}10^{-4}$) gives surface heat flow 62.9 mW m^{-2} and a Moho temperature of 807.4°C. Using granite parameters ($A = 0.2$; $B = 4.1\text{x}10^{-4}$) yields a surface heat flow of 78 mW m^{-2} and Moho temperature 809.7°C, with all other parameters held fixed. This is an interesting result in that it suggests surface heat flow variation partly

reflects the crustal compositional effect on thermal conductivity, and this may help to explain a correlation of v_P/v_S with surface heat flow described in Lowry & Pérez-Gussinyé (2011) (which that paper inferred to be a negligible effect). Also interestingly, the resulting difference in Moho temperature is negligible.

The geotherms and their differences are plotted above. Interestingly, the geotherms and gradients are almost identical through the crust, but the higher heat flow through granitic crust (blue curve) results in slightly lower (by a few tens of degrees) temperatures in the upper mantle of continental crust, with greatest differences at depths typical of mantle partial melting. Note that these effects (i.e., higher heat flow and changed mantle temperature) may not be "real" however in that we have artificially specified identical fixed thermal length-scales l_{con} for the crust. In effect, we changed conductivity $k = 1/(A + BT)$ without correspondingly changing the thermal diffusivity κ (which would in turn change l_{con}), and that forced the heat flow $q_s = -k\, \mathrm{d}T/\mathrm{d}z$ to change. The identical crustal thermal length-scales force different thermal length-scales in the mantle to meet the requirement that heat flow is continuous across the Moho. In reality, the lithosphere may thermally equilibrate with different crustal thermal length-scales given different crustal thermal conductivities.

(b) What are the effects of higher versus lower crustal heat production (H_0 or l_{rad})?

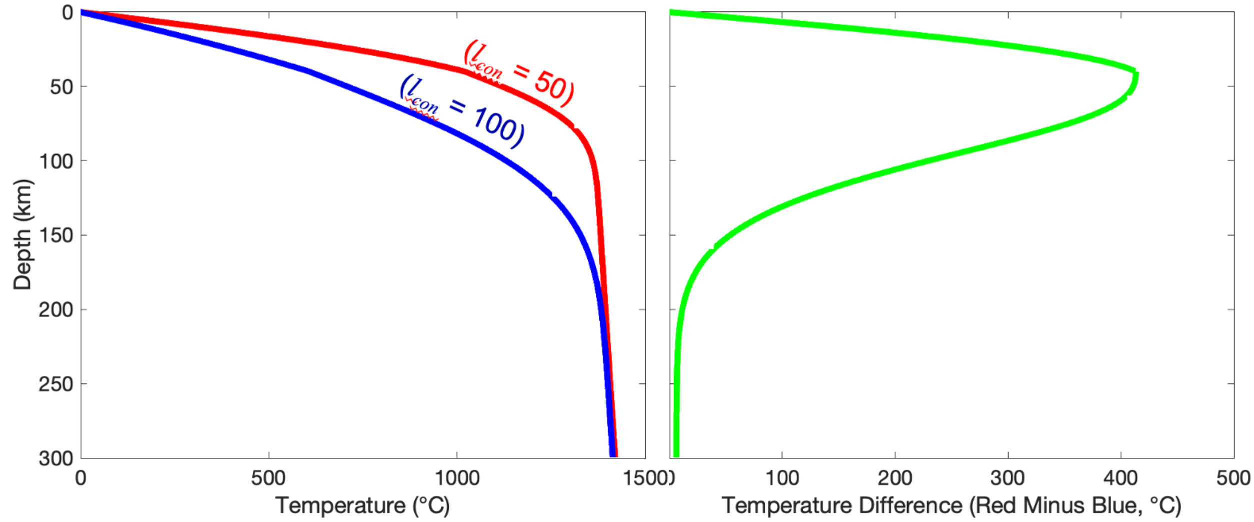
[5] I changed only H_0, using 2.0 μW m^{-3} for a high (granitic = red) end member, and 0.5 μW m^{-3} for low (basaltic = blue). The effects on surface heat flow were similar to that of using different crustal thermal conductivities, resulting in a 75.9 mW m^{-2} (granitic) versus 63.5 mW m^{-2} (basaltic) surface heat flow. The effects on crustal and Moho temperature were larger, resulting in 832°C (granitic) versus 799.1°C (basaltic) Moho temperature, with a peak difference in geotherm ~37°C near the midcrust.



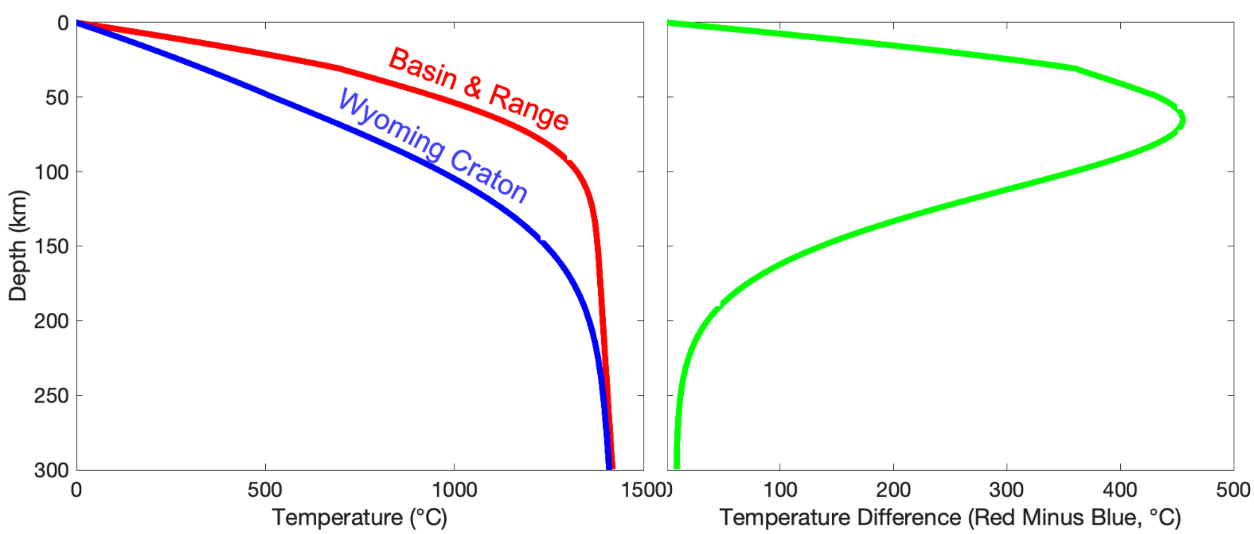
(c) What are the effects of higher versus lower conductive lengthscale l_{con}?

[5] Changing l_{con} within reasonable ranges can have a much larger effect on the geotherm than changing the other parameters. Just changing l_{con} from 100 (blue) to 50 km (red)

results in an increase in surface heat flow from 50.3 to 90.8 mW m^{-2} and in Moho temperature from 607.9°C to 1021.2°C. The greatest temperature difference is around 410°C in the uppermost mantle.



(d) The Basin and Range in the Bonneville region has surface heat flow ~90 mW/m², A_0 ~1.3 μW/m³, crustal thickness 31 km, and Moho temperature ~700°C. The Wyoming craton nearby has surface heat flow ~60 mW/m², A_0 ~1.5 μW/m³, crustal thickness 48 km, and Moho temperature ~500°C. First, given these, find a geotherm parameterization that matches these observations. (Describe the parameters you used, and include the plot of the two geotherms together that is created as a .png file by the Matlab script). Do the parameters and geotherms seem reasonable? Note that l_{con} has a physical meaning! It reflects either advective heat transfer or transient cooling/heating, and it can be related to steady-state extensional strain rate using Lowry et al. (J. Geophys. Res., 2000) eqn 6, or with half-space cooling (compare Lowry et al. (2000) eqn 5 with T&S eqn 4-113). Are these results consistent with what you might expect for these regions?



[10] Using the Basin & Range H_0 and crustal thickness parameters plus a ~granitic thermal conductivity ($A = 0.175$; $B = 4.2\text{x}10^{-4}$), $l_{rad} = 8$ km and $l_{con} = 64.8$ km gives surface heat flow $q_s = 90.0$ mW m^{-2} and Moho temperature of 699.2 °C. Using the Wyoming craton H_0 and crustal thickness with a quartz-rich granitic low-temperature thermal conductivity ($A = 0.11$; $B = 4\text{x}10^{-4}$), $l_{rad} = 10$ km and $l_{con} = 151$ km gives surface heat flow $q_s = 59.8$ mW m^{-2} and Moho temperature of 502.2 °C. The geotherms have a maximum difference of 455 °C at about 65 km depth and are plotted above.

A quartz-rich crust in both regions is reasonable based on v_P/v_S imaging, and it is reasonable to expect that radioactive heating would extend a little deeper in the Wyoming craton than in the Basin & Range, as suggested by differences in l_{rad}, given the history of extension in the latter province. The discrepancy of surface heat flow and Moho temperatures, given more physically reasonable rock thermal parameters, indicates that other processes besides conductive thermal transfer are at play, as has been discussed in class. The primary source of differences in the two geotherms is in l_{con}, which is much greater in the Wyoming craton than in the Basin & Range. This is consistent with both the advective heating and transient cooling mechanisms for varying l_{con}: The Basin and Range is tectonically younger than the Wyoming craton in that it has had much more recent thermal perturbation by emplacement of melts in the crust (a process which may still be ongoing), and it is also actively extending so has much higher extensional strain rate than the craton.

Grads only (but undergrads can do for extra credit!)
(6) The temperature differences in your two geotherms in part 5 should result in an isostatic elevation difference for the two locations. Modify the Matlab script Geotherm.m to calculate the Airy isostatic difference in elevation expected for these two cases. How does that compare with observed elevations?

[5] Similar to questions 2 and 3, we want $\rho_0 g \Delta h = \int \rho_0 g \alpha \Delta T \mathrm{d}z$. To calculate this, one should first recognize that the geotherms are calculated at 100 m intervals (`dz=100;` in line 13 of the script), and the two geotherms are stored in `tkp` and `tmp`. With that information, and recognizing that ρ_0 and g cancel in the equation above, one can calculate the elevation difference with a single line placed near the end of the script:

```
DeltaH=dz*2.5e-5*sum(tkp-tmp)
```

(Here, using $\alpha = 2.5\text{x}10^{-5}$ °C^{-1} remains consistent with the value assumed in Roy et al. (2009) and used earlier in the homework set). For the geotherm parameters that I used, described in 5d above, this calculation gives a thermal elevation of the Basin and Range that is 1.3 km higher (!) than that of the Wyoming craton. Of course, that difference in thermal elevation is mostly (if not completely!) offset by the differences in crustal thickness of the two provinces.