GEO5/6670 Inverse Theory        Assignment II        Due: May 5 2021

For the first problem we'll use A0_Qs_scatter.xy, the file of surface radiogenic heat production versus surface heat flow we used in Assignment I. Recall that the file format is surface radiogenic heat production ($\mu$W m$^{-3}$) in *column 1*; surface heat flow (mW m$^{-2}$) in *column 2*; and the standard deviation of surface heat flow in *column 3* (mW m$^{-2}$).

1. Building on what we did for Asgt I, write a matlab (*.m) script to invert for mantle heat flow $Q_m$ and radiogenic length parameter $l_{rad}$ using stochastic inversion. (Be sure to think about units! Convert numbers in your file to mks for simplicity.) Use data covariance weighting as implied in column 3 of your file, and describe the model parameter expected value and covariance matrix using two different approaches:
(a) Assume the a priori expectation of mantle heat flow is 18 ± 3 mW m$^{-2}$ (loosely based on Jaupart & Mareschal, *Treatises on Geophysics*, 2007) and the a priori expectation of the lengthscale of radiogenic heat production is 9.4 ± 5 km (an even rougher approximation based on distributions of Sierra Nevada xenoliths in Brady et al., *Lithos*, 2006).
(b) Now, substitute the a posteriori parameter covariance matrix from your earlier (Assignment 1) foray into weighted least squares inversion, using the same expected values for model parameters as in part a.

Your matlab script should print all of the results to the screen in a clear, well-labeled fashion, and include solution appraisal of the types that we have already discussed and used in class (e.g., $\chi^2$ parameters of fit; formal model parameter uncertainties). Compare these results to those derived earlier from OLS & WLS and to each other. *Do you think these estimates tell us more about the modeling problem and the uncertainties in model parameters than what we learned earlier from OLS and WLS?*

For the rest of the assignment, we will perform nonlinear inversion of GPS time series data for parameters of a transient fault slip event. Download the file N_CAYA.dat from the course website. This contains date (decimal year) in *column 1*; north position (cm) in *column 2*; and standard error (cm) in *column 3*. Note that some of the (daily averaged) positions in the file are missing (i.e, there are gaps in the data). We will use these measurements as the observed data to fit a function describing transient deformation of the form:

$$x(t) = x_0 + Vt + \frac{\Delta x}{2}\left[\tanh\frac{t-T_0}{\tau} - 1\right] \qquad (1)$$

Note that this model is strictly nonlinear only in terms of the parameters $T_0$, the midtime of the transient, and $\tau$, the timescale parameter of transient displacement. The rest of the parameters: Position at time zero, $x_0$; velocity $V$; and displacement during the transient event, $\Delta x$, can be related linearly to the data and so we will treat this as a mixed linear & nonlinear problem. There are three linear parameters and a total of five different parameters to solve for in this problem.

We want to solve for the three linear parameters and two timescale parameters as a single problem with two steps (one linear; one nonlinear). First subtract the mean from the time vector (this stabilizes the problem; otherwise $x_0$ would be much larger than the other parameters!) but save the mean time you subtracted so that you can add it back to your results later.

2. Find a best fit $T_0$ and $\tau$ using a grid search coupled with a weighted least squares solution for the linear parameters (i.e., given fixed $T_0$ and $\tau$, solve the linear WLS problem for the other nine parameters at each point in the grid search). Use the WLS error norm and estimate the error contours from $E_{min}$ and the likelihood ratio method. Do the 1-sigma and 2-sigma confidence intervals show evidence of the nonlinearity of the problem? Do the $\chi^2$ parameter of misfit and the model parameter covariance estimates derived from the sensitivity matrix make sense? Plot the best-fit model and data together. Does the fit look reasonable?

3. Now solve the nonlinear portion of the problem again using iterative nonlinear inversion with the gradient method (the Taylor-series approximation), using a starting model with $T_0$ = 2000.0 and $\tau$ = 10 days. Does this give a significantly different result?

4. Finally, write and implement a simulated annealing inversion for the nonlinear parameters, using the same starting model as above. How does this result compare to the other two?

You should send me (as separate email attachments) all of your matlab scripts, and a .pdf file that contains all relevant plots and descriptions of what you did– along with your results and your answers to each of the discussion questions– by class-time on Wednesday, May 5.